Pseudocompact frames *L* versus different topologies on *R (L)*

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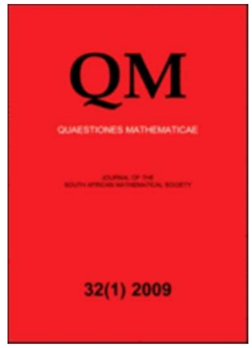

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Pseudocompact frames L versus different topologies on $\mathcal{R}(L)$

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Abstract. *In this paper we have characterized pseudocompact frames L (1) via u-topology and m-topology on the rings $\mathcal{R}(L)$ and $\mathcal{R}^*(L)$; (2) via some special kind of ideals of $CozL$.*

1. Introduction

We initiate this paper after clearly stating that each frame L that will appear in this article will be assumed to be completely regular. Our main intention is to characterize pseudocompact frames L via the rings $\mathcal{R}(L)$ and $\mathcal{R}^*(L)$ equipped with the u-topology and the m-topology. Here $\mathcal{R}(L)$ and $\mathcal{R}^*(L)$ are respectively commutative lattice ordered rings of all frame maps from the frame $\mathcal{L}(\mathbb{R})$ of reals to L and that of all bounded frame maps from $\mathcal{L}(\mathbb{R})$ to L. For further details about these rings, see Banaschewski [**2**]. A number of characterizations of these frames in terms of some corresponding algebraic properties of these rings have already been given by Dube and Matutu (see [**4**] and [**5**]), Dube (see [**8**] and [**9**]) and Banaschewski and Gilmour (see [**3**]). We have shown in this paper that, a frame L is pseudocompact if and only if the set U of all multiplicative units of $\mathcal{R}(L)$ is open in the u-topology if and only if $\mathcal{R}(L)$ with u-topology is a topological ring if and only if $\mathcal{R}(L)$ with u-topology is a topological vector space over $\mathbb{Q}$ (Theorem 3.7) if and only if the relative m-topology on $\mathcal{R}^*(L)$ and the u-topology on $\mathcal{R}^*(L)$ coincide (Theorem 3.8). These results are pointfree analogue of the corresponding characterizations of pseudocompact topological spaces (see [**10**, 2M and 2N]). However it seems worth mentioning that Hewitt [**11**] has incorrectly written in his monumental paper on Rings of Continuous Functions long time back in 1948 that, $C(X)$ with u-topology is always a topological vector space, irrespective of whether or not X is pseudocompact and the same error has been carried on in the 1972 paper of Nanzetta and Plank [**12**]. These last two authors have offered a characterization of pseudocompact spaces in the manner that X is pseudocompact if and only if the closure of any ideal in $C(X)$ is an ideal if and only if each ideal in $C(X)$ is contained in a closed ideal, $C(X)$ being equipped with the u-topology (see [**12**, Theorem 2.1]); in this paper by an ideal the authors have meant a proper ideal in the corresponding ring. We have achieved the pointfree version of this result too in the present paper (Theorem 3.10) and have also understood an ideal in $\mathcal{R}(L)$ or $\mathcal{R}^*(L)$ to be a proper ideal. Furthermore we have shown that if a frame L has the *pretty property* defined by Dube (see [**7**, page 127]) in the manner that, for each $f \in \mathcal{R}(L)$ and $u \in U^+$, the set of all positive units of $\mathcal{R}(L)$, there exists $g \in \mathcal{R}(L)$ such that $cozg \prec\prec cozf$

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and $|g-f| \leq u$, then L is pseudocompact if and only if every closed ideal of $\mathcal{R}^*(L)$ in the m-topology inherited from $\mathcal{R}(L)$ is the intersection of all maximal ideals of $\mathcal{R}^*(L)$ containing it (Theorem 3.11). Finally we have shown on using Axiom of Choice (AC) that, a frame L is pseudocompact if and only if every ideal of $CozL$ is σ-proper (Theorem 4.1), which is the pointfree analogue of the following classical result: a topological space X is pseudocompact if and only if every z-filter has the countable intersection property (see [**10**, 5H]).

2. Preliminaries

For general theory of frames and the ring of all real valued continuous functions on frames, we refer [**1**], [**2**] and [**13**]. Nevertheless, in spite of repetitions let us explain the meaning of a few notations, which we will use in this article from time to time. $\mathbb{Q}^+$ will stand for the set of all positive rational numbers, for any $r \in \mathbb{Q}$, $\mathbf{r}$ will mean the corresponding constant map in $\mathcal{R}(L)$. Also for any $p \in \mathbb{Q}$, $(-,p)$ and $(p,-)$ will stand for respectively $\bigvee_{r\in\mathbb{Q}}(r,p)$ and $\bigvee_{q\in\mathbb{Q}}(p,q)$ in the frame $\mathcal{L}(\mathbb{R})$ of reals. Let βL, the set of all regular ideals of L, be the Stone-Čech compactification of L and $\Sigma\beta L$ be the set of all prime elements of βL. Then for $I \in \beta L$ we use the notations, $M^I = \{f \in \mathcal{R}(L) : r(cozf) \subseteq I\}$ and $M^{*I} = \{f \in \mathcal{R}^*(L) : coz(f^\beta) \subseteq I\}$, r standing for the right adjoint of the join map $j : \beta L \to L$ and $f^\beta : \mathcal{L}(\mathbb{R}) \to \beta L$ is the frame extension of $f \in \mathcal{R}^*(L)$ (see [**6**] and [**9**]).

Definition 2.1. A frame L is called *pseudocompact* if $\mathcal{R}(L) = \mathcal{R}^*(L)$.

Definition 2.2. Set for any $f \in \mathcal{R}(L)$ and $r \in \mathbb{Q}^+$, $u(f,r) = \{g \in \mathcal{R}(L) : |f-g| \leq \mathbf{r}\}$. Then there is a unique topology on $\mathcal{R}(L)$ for which for any $f \in \mathcal{R}(L)$, the family $\{u(f,r) : r \in \mathbb{Q}^+\}$ forms a base for the neighbourhood system of f. We call this topology as in the classical case for $C(X)$, the *u-topology* on $\mathcal{R}(L)$. A typical basic neighbourhood in the u-topology on the subring $\mathcal{R}^*(L)$ of $\mathcal{R}(L)$ will be denoted by $u^*(f,r)$, $f \in \mathcal{R}^*(L)$.

Definition 2.3. Set for any $f \in \mathcal{R}(L)$ and $u \in U^+$, $m(f,u) = \{g \in \mathcal{R}(L) : |f-g| \leq u\}$. Then there is a unique topology on $\mathcal{R}(L)$ for which for any $f \in \mathcal{R}(L)$, the family $\{m(f,u) : u \in U^+\}$ forms a base for the neighbourhood system of f. We call this topology as in the classical situation, the *m-topology* on $\mathcal{R}(L)$.

3. Pseudocompact frames L via u-topology and m-topology on $\mathcal{R}(L)$ and $\mathcal{R}^*(L)$.

Lemma 3.1. *An $f \in \mathcal{R}(L)$ is a unit of $\mathcal{R}(L)$ if and only if $cozf = 1$.*

Proof. See [**1**, Proposition 3.3.1]. □

Lemma 3.2. *For an $f \in \mathcal{R}(L)$, the following are equivalent:*
(1) *f is a unit of $\mathcal{R}^*(L)$.*
(2) *there exists $p \in \mathbb{Q}^+$ such that $f(-,-p) \vee f(p,-) = 1$.*
(3) *there exists $p \in \mathbb{Q}^+$ such that $|f| \geq \mathbf{p}$.*

Proof. (1) ⇒ (2): By hypothesis there exists $g \in \mathcal{R}^*(L)$ such that $fg = 1$ and of course there exists $m \in \mathbb{N}$ such that $g(-m,m) = 1$. This yields in view of a well known formula (see [**1**, Proposition 3.3.1]) that, $f(-,-\frac{1}{m}) \vee f(\frac{1}{m},-) = g(-m,m) = 1$.
(2) ⇒ (3): It is sufficient to show in view of a result of Banaschewski (see [**2**, Lemma 4]) that $|f|(q,-) \geq \mathbf{p}(q,-)$, for each $q \in \mathbb{Q}$. If $p \leq q$ then $|f|(q,-) \geq 0 = \mathbf{p}(q,-)$. Again if $q < p$ then $f(q,-) \geq f(p,-)$ and $f(-,-q) \geq f(-,-p)$ implying that, $|f|(q,-) \geq f(-,-q) \vee f(q,-) = 1 = \mathbf{p}(q,-)$.

(3) $\Rightarrow$ (2): Let $p \in \mathbb{Q}^+$ such that $|f| \geq \mathbf{p}$. Then $1 = |f|(\frac{p}{2}, -) = (f \vee (-f))(\frac{p}{2}, -) \leq f(\frac{p}{2}, -) \vee (-f)(\frac{p}{2}, -) = f(\frac{p}{2}, -) \vee f(-, -\frac{p}{2})$. Hence $f(\frac{p}{2}, -) \vee f(-, -\frac{p}{2}) = 1$.
(2) $\Rightarrow$ (1): Let $p \in \mathbb{Q}^+$ such that $f(-, -p) \vee f(p, -) = 1$. Then $coz f = f(-, 0) \vee f(0, -) = 1$ and hence by Lemma 3.1 there exists $g \in \mathcal{R}(L)$ such that $fg = 1$ and $g(-\frac{1}{p}, \frac{1}{p}) = f(-, -p) \vee f(p, -) = 1$. So $g \in \mathcal{R}^*(L)$ and therefore f is a unit of $\mathcal{R}^*(L)$. □

Lemma 3.3. *Let U^* be the set of all units of $\mathcal{R}^*(L)$. Then U^* is an open subset of $\mathcal{R}^*(L)$ in the u-topology.*

Proof. Choose $u \in U^*$. Then by Lemma 3.2 there exists $p \in \mathbb{Q}^+$ such that $|u| \geq \mathbf{p}$. Now the set $E = \{f \in \mathcal{R}^*(L) : |f - u| \leq \frac{\mathbf{p}}{\mathbf{2}}\}$ is a neighbourhood of u each member of which is surely a unit of $\mathcal{R}^*(L)$. Thus u is an interior point of U^* and hence U^* is open. □

Lemma 3.4. *$\mathcal{R}^*(L)$ is a topological ring as well as a topological vector space over $\mathbb{Q}$ with respect to the u-topology.*

Proof. We have to show that the addition and the multiplication on $\mathcal{R}^*(L)$ are continuous. So let $f, g \in \mathcal{R}^*(L)$, $r \in \mathbb{Q}^+$, $u^*(f+g, r)$ and $u^*(fg, r)$ be arbitrary neighbourhoods of $f+g$ and fg respectively. Then $u^*(f, \frac{r}{2})$ and $u^*(g, \frac{r}{2})$ are neighbourhoods of f and g respectively and $u^*(f, \frac{r}{2}) + u^*(g, \frac{r}{2}) \subseteq u^*(f + g, r)$. Since $f, g \in \mathcal{R}^*(L)$, there exists $n, m \in \mathbb{N}$ such that $|f| \leq \mathbf{n}$ and $|g| \leq \mathbf{m}$. It is not hard to check that, $u^*(f, \frac{r}{2(\frac{r}{2n}+m)}).u^*(g, \frac{r}{2n}) \subseteq u^*(fg, r)$. □

Lemma 3.5. *If L is not pseudocompact then the set U of all units of $\mathcal{R}(L)$ is not an open subset of $\mathcal{R}(L)$.*

Proof. Since L is not pseudocompact, there exists a positive unit f of $\mathcal{R}(L)$ such that f is not a unit of $\mathcal{R}^*(L)$. Hence by Lemma 3.2, $f(-, -r) \vee f(r, -) \neq 1$ for any $r \in \mathbb{Q}^+$. Again we see that for any $r \in \mathbb{Q}^+$, the function $(f - \mathbf{r}) \vee \mathbf{0}$ belongs to $u(f, r)$ simply because $|f - ((f - \mathbf{r}) \vee \mathbf{0})| = |\mathbf{r} \wedge f| \leq \mathbf{r}$, but this function does not belong to U as $coz((f - \mathbf{r}) \vee \mathbf{0}) = f(r, -)$ (see [**2**, Lemma 6]) $= f(-, -r) \vee f(r, -)$ (as $f \geq \mathbf{0}$) $\neq 1$. Therefore $f \in U$, is not an interior point of U and hence U is not open. □

Lemma 3.6. *If L is not pseudocompact then $\mathcal{R}(L)$ is neither a topological ring nor a topological vector space over $\mathbb{Q}$ with respect to the u-topology.*

Proof. Since L is not pseudocompact, there exists $f \in \mathcal{R}(L) - \mathcal{R}^*(L)$. We shall show that the multiplication on $\mathcal{R}(L)$ is not continuous at the point $(\mathbf{0}, f)$. Indeed the set $S = \{g \in \mathcal{R}(L) : |g| \leq \mathbf{1}\}$ is a neighbourhood of $\mathbf{0}$ in $\mathcal{R}(L)$. Now for any neighbourhood $u(\mathbf{0}, r)$ of $\mathbf{0}$ and $u(f, s)$ of f in $\mathcal{R}(L)$, it is not hard to check that $u(\mathbf{0}, r).u(f, s) \nsubseteq S$, because the function $\frac{\mathbf{r}}{\mathbf{2}}.f \in u(\mathbf{0}, r).u(f, s)$ but $\frac{\mathbf{r}}{\mathbf{2}}.f \notin S$. For otherwise $|\frac{\mathbf{r}}{\mathbf{2}}.f| \leq 1$ implies, $(\frac{\mathbf{r}}{\mathbf{2}}.f)(-, -1) = 0 = (\frac{\mathbf{r}}{\mathbf{2}}.f)(1, -)$, which in conjunction with the relation $(-, -1) \vee (-2, 2) \vee (1, -) = 1_{\mathcal{L}(\mathbb{R})}$ implies that, $1 = (\frac{\mathbf{r}}{\mathbf{2}}.f)(-2, 2) = \bigvee\{\frac{\mathbf{r}}{\mathbf{2}}(p, q) \wedge f(t, s) : \langle p, q\rangle.\langle t, s\rangle \subseteq \langle -2, 2\rangle\} = \bigvee\{f(t, s) : \langle p, q\rangle.\langle t, s\rangle \subseteq \langle -2, 2\rangle$ and $p < \frac{r}{2} < q\} \leq f(-\frac{4}{r}, \frac{4}{r})$ and hence $f(-\frac{4}{r}, \frac{4}{r}) = 1$ which contradicts the fact that f is unbounded.
Almost analogous argument can be adapted to show that the scalar multiplication: $\mathbb{Q} \times \mathcal{R}(L) \to \mathcal{R}(L)$ is not continuous at the point $(0, f)$. □

THEOREM 3.7. *For a frame L, the following are equivalent:*
(1) *L is pseudocompact.*
(2) *U is an open subset of $\mathcal{R}(L)$ in the u-topology.*
(3) *$\mathcal{R}(L)$ with u-topology is a topological ring.*
(4) *$\mathcal{R}(L)$ with u-topology is a topological vector space over $\mathbb{Q}$.*

PROOF. Follows from Lemma 3.3, Lemma 3.4, Lemma 3.5 and Lemma 3.6. □

THEOREM 3.8. *For a frame L, the following are equivalent:*
(1) *L is pseudocompact.*
(2) *the u-topology and the relative m-topology on $\mathcal{R}^*(L)$ coincide.*

PROOF. It is easy to see that the u-topology on $\mathcal{R}^*(L)$ is weaker than the relative m-topology on $\mathcal{R}^*(L)$.
(1) $\Rightarrow$ (2): Let L be pseudocompact. Then any positive unit u of $\mathcal{R}(L)$ is also a positive unit of $\mathcal{R}^*(L)$ and so by Lemma 3.2, there exists $p \in \mathbb{Q}^+$ such that $\mathbf{p} \leq u$. Therefore for any $f \in \mathcal{R}^*(L)$, $u(f,p) = m(f,p) \subseteq m(f,u)$ and hence the u-topology on $\mathcal{R}^*(L)$ is finer than the relative m-topology on $\mathcal{R}^*(L)$. Therefore these two topologies are identical.
(2) $\Rightarrow$ (1): Let L be not pseudocompact. It is enough to show in view of Lemma 3.4 that $\mathcal{R}^*(L)$ is not a topological vector space over $\mathbb{Q}$ with relative m-topology. Since L is not pseudocompact, there exists a positive unit u of $\mathcal{R}(L)$ which is not a unit of $\mathcal{R}^*(L)$ and hence by Lemma 3.2, $\mathbf{p} \nleq u$ for any $p \in \mathbb{Q}^+$. Therefore for any pair of distinct rational numbers r, s it will never happen that $|\mathbf{r} - \mathbf{s}| \leq u$. Accordingly for any $r \in \mathbb{Q}$, $m(\mathbf{r}, u) \cap \{\mathbf{s} : s \in \mathbb{Q}\} = \{\mathbf{r}\}$-in other words the set $\{\mathbf{r} : r \in \mathbb{Q}\}$ of constant functions is a discrete subset of $\mathcal{R}^*(L)$. Therefore the scalar multiplication: $\mathbb{Q} \times \mathcal{R}^*(L) \to \mathcal{R}^*(L)$ is not continuous at points $(r, \mathbf{s})$, $r, s \in \mathbb{Q}$ with $r \neq s$. □

LEMMA 3.9. *In any topological ring A, the closure of an ideal I is either a proper ideal or the whole of A. In particular as is evident from Lemma 3.3 and Lemma 3.4 that the closure of each ideal in $\mathcal{R}^*(L)$ with u-topology is also an ideal.*

PROOF. See [**10**, 2M]. □

THEOREM 3.10. *For a frame L, the following are equivalent:*
(1) *L is pseudocompact.*
(2) *The closure in u-topology of any ideal in $\mathcal{R}(L)$ is an ideal.*
(3) *Each ideal in $\mathcal{R}(L)$ with u-topology is contained in a closed ideal.*

PROOF. (1) $\Rightarrow$ (2): Follows from Lemma 3.4 and Lemma 3.9.
(2) $\Rightarrow$ (3): Clear.
(3) $\Rightarrow$ (1): Suppose L is not pseudocompact. Then there exists $f \in \mathcal{R}(L) - \mathcal{R}^*(L)$ such that f is positive unit of $\mathcal{R}(L)$. Consequently for each $n \in \mathbb{N}$, each $a_n = f(-,n)$ is strictly less than 1 in L. Therefore $I = \{g \in \mathcal{R}(L) : coz g \leq a_n$ for some $n\}$ is an ideal of $\mathcal{R}(L)$ and hence by hypothesis contained in a closed ideal say, J. We shall show that $\frac{1}{f} \in J$ and this will contradict the fact that J is an ideal. Indeed for any $r \in \mathbb{Q}^+$ choose a positive integer n such that $\frac{1}{n} \leq r$. Define $g = (\frac{1}{f} - \frac{\mathbf{1}}{\mathbf{n}}) \vee \mathbf{0}$. Then $coz g = \frac{1}{f}(\frac{1}{n}, -)$ (see [**2**, Lemma 6]) $\leq f(-,n) = a_n$, so that $g \in I \subseteq J$, but $|\frac{1}{f} - g| = |\frac{\mathbf{1}}{\mathbf{n}} \wedge \frac{1}{f}| \leq \frac{\mathbf{1}}{\mathbf{n}} \leq \mathbf{r}$. Hence $\frac{1}{f} \in \bar{J} = J$. □

THEOREM 3.11. *Let L have the pretty property. Then the following are equivalent:*
(1) *L is pseudocompact.*
(2) *every closed ideal of $\mathcal{R}^*(L)$ in the relative m-topology is the intersection of all maximal ideals of $\mathcal{R}^*(L)$ containing it.*

PROOF. (1) ⇒ (2): Follows from Lemma 3.20 of [**7**].
(2) ⇒ (1): Let the condition (2) be true. To show that (1) is true, it is sufficient to show that every maximal ideal of $\mathcal{R}(L)$ is real (see [**8**, Proposition 4.4]). So let $M^I(I \in \Sigma\beta L)$ be a maximal ideal of $\mathcal{R}(L)$ (see [**6**, Proposition 5.1]). Then $M^I \cap \mathcal{R}^*(L)$ is a prime ideal of $\mathcal{R}^*(L)$ and hence is contained in a unique maximal ideal of $\mathcal{R}^*(L)$ as because $\mathcal{R}^*(L) \cong \mathcal{R}(\beta L)$ and $\mathcal{R}(L)$ is a Gelfand ring for any frame L (see [**6**, Proposition 5.4]). On the other hand it follows from Lemma 4.1 of [**9**] that, $M^I \cap \mathcal{R}^*(L) \subseteq M^{*I}$ and M^{*I} is a maximal ideal of $\mathcal{R}^*(L)$ (see [**9**, Proposition 3.8]). Since every maximal ideal of $\mathcal{R}(L)$ is closed in m-topology (see [**7**, Lemma 3.19]), $M^I \cap \mathcal{R}^*(L)$ is a closed ideal of $\mathcal{R}^*(L)$ and so by hypothesis, it is the intersection of maximal ideals of $\mathcal{R}^*(L)$ containing it. Hence $M^I \cap \mathcal{R}^*(L) = M^{*I}$. Therefore by Proposition 4.2 of [**9**] and Corollary 3.7 of [**8**], it follows that M^I is a real maximal ideal of $\mathcal{R}(L)$. □

4. Pseudocompact frames L via ideals of $CozL$.

THEOREM 4.1. <u>(AC)</u>: *For a frame L, the following are equivalent:*
(1) *L is pseudocompact.*
(2) *every ideal I of $CozL$ is σ-proper in the sense that for any countable subset $S \subseteq I$, $\bigvee S \neq 1$.*

PROOF. (1) ⇒ (2): Let L be pseudocompact. Then every maximal ideal of $\mathcal{R}(L)$ is real (see [**8**, Proposition 4.4]). Let I be an ideal of $CozL$ and J, a maximal ideal of $CozL$ with $I \subseteq J$. Then $Coz^{\leftarrow}[J] = \{f \in \mathcal{R}(L) : coz f \in J\}$ is a maximal ideal of $\mathcal{R}(L)$ (see [**6**, page 157]) and hence $J = Coz[Coz^{\leftarrow}[J]]$ is σ-proper (see [**8**, Proposition 3.6]). Therefore I is σ-proper as $I \subseteq J$.
(2) ⇒ (1): Let the condition (2) hold. To show L is pseudocompact, it is sufficient to show that every maximal ideal of $\mathcal{R}(L)$ is real (see [**8**, Proposition 4.4]). So let M be a maximal ideal of $\mathcal{R}(L)$. Then $Coz[M] = \{coz f : f \in M\}$ is an ideal of $CozL$ (see [**6**, page 157]) and hence by hypothesis $Coz[M]$ is σ-proper and therefore M is a real ideal of $\mathcal{R}(L)$ (see [**8**, Proposition 3.6]). □

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